ON THE QUASI-ONE-DIMENSIONAL STEADY FLOW OF A COMPRESSIBLE CONDUCTING GAS IN A PIPE OF CONSTANT CROSS-SECTION IN THE PRESENCE OF TRANSVERSE MAGNETIC AND ELECTRIC FIELDS

(O KVAZIODOMERNOM STATSIONARNOM TECHENII CZHIMAEMOGO PROVODIASHCHEGO GAZA V KANALE POSTOIANNOGO SECHENIIA PRI NALICHII POPERECHNYKH MAGNITNOGO I ELEKTRICHESKOGO POLEI)

PMM Vol.24, No.3, 1960, pp. 546-547

I. B. CHEKMAREV
(Leningrad)

Quasi-one-dimensional steady flow of a compressible conducting gas in the presence of transverse magnetic and electric fields was considered by Resler and Sears in their papers [1,2]. In the present note the equations of quasi-one-dimensional steady flow of a conducting gas are integrated in the particular case when the intensity of the external electric field is proportional to the intensity of the magnetic field.

Let us assume that the gas moves parallel to the axis of x, the magnetic field is parallel to the axis of y, and the electric field to the axis of z. Viscosity and heat conduction of the gas are neglected. The intensity of the external electric field is proportional to the intensity of the magnetic field and varies according to the law $E = -\mu u_m H$, where u_m is the constant coefficient of proportionality. For the units of velocity of the gas u, pressure p, density ρ, temperature T and intensity of the magnetic field H let us choose their values u_0, p_0, ρ_0, T_0, H_0 at the section $x = 0$. For the unit of the coordinate x let us choose the characteristic dimension of the pipe L. Then the basic equations of the problem in dimensionless form are

$$\rho u \frac{du}{dx} + \frac{1}{kM_0^2}\frac{dp}{dx} + SH\frac{dH}{dx} = 0, \qquad \frac{d}{dx}\rho u = 0 \tag{1}$$

$$\frac{1}{k-1}\rho u \frac{dT}{dx} + p\frac{du}{dx} = \frac{SkM_0^2}{R_m}\left(\frac{dH}{dx}\right)^2, \qquad p = \rho T, \qquad \frac{dH}{dx} = R_m (u - u_m) H. \tag{2}$$

where

$$M_0^2 = \frac{u_0^2}{kp_0/\rho_0}, \qquad k = \frac{C_p}{C_v}, \qquad S = \frac{\mu H_0^2}{\rho_0 u_0^2}, \qquad R_m = \sigma\mu u_0 L$$

The dimensionless boundary conditions have the form

$$u = p = \rho = T = H = 1 \quad \text{when} \quad x = 0 \tag{3}$$

From (1) we obtain two integrals of the system:

$$u + \frac{1}{kM_0^2} p + \frac{SH^2}{2} = h, \qquad \rho u = 1 \tag{4}$$

Multiplying the first of Equations (1) by u and adding it to the first of Equations (2), we obtain

$$\frac{d}{dx}\left[\frac{k}{k-1} u \left(h - \frac{SH^2}{2} - u\right) + \frac{u^2}{2}\right] = -SR_m u_m (u - u_m) H^2 \tag{5}$$

where the temperature T and the pressure p have been eliminated with the help of the second Equation (2) and the first Equation (4). Multiplying the third Equation (2) by H and using this to transform the right-hand side of (5), we find yet another integral of the system

$$-\frac{k+1}{2(k-1)} u^2 + \frac{k}{k-1} hu - \left(\frac{k}{k-1} u - u_m\right)\frac{SH^2}{2} = g \tag{6}$$

Integrating the third Equation (2), we have

$$H = \exp\left[R_m \int_0^x (u - u_m)\, dx\right] \tag{7}$$

Eliminating the intensity of the magnetic field from (6) and (7), we obtain the equation

$$\exp\left[2R_m \int_0^x (u - u_m)\, dx\right] = -\frac{2}{S} \frac{\frac{k+1}{2(k-1)} u^2 - \frac{k}{k-1} hu + g}{\frac{k}{k-1} u - u_m} \tag{8}$$

Introducing the characteristic velocities

$$u_{1,2} = \frac{kh \pm \sqrt{(kh)^2 - 2(k^2-1)g}}{k+1}$$

let us transform Equation (8) to the form

$$\exp\left[2R_m \int_0^x (u - u_m)\, dx\right] = -\frac{k+1}{S} \frac{(u - u_1)(u - u_2)}{k(u - u_m) + u_m} \tag{9}$$

Taking logarithms and differentiating (9), we find that

$$2R_m\, dx = \frac{[(u - u_1) + (u - u_2)][k(u - u_m) + u_m] - k(u - u_1)(u - u_2)}{[k(u - u_m) + u_m](u - u_1)(u - u_2)(u - u_m)}\, du \tag{10}$$

Integrating (10) and making use of the boundary condition for velocity, we have

$$2R_m x = \frac{1}{u_1 - u_m} \ln \frac{(u - u_1)(1 - u_m)}{(u - u_m)(1 - u_1)} + \frac{1}{u_2 - u_m} \ln \frac{(u - u_2)(1 - u_m)}{(u - u_m)(1 - u_2)} + \\ + \frac{k}{u_m} \ln \frac{[k(u - u_m) + u_m](1 - u_m)}{[k(1 - u_m) + u_m](u - u_m)} \qquad (11)$$

For the intensity of the magnetic field H, on taking account of (7) and (9), we obtain the formula

$$H = \sqrt{-\frac{k+1}{S} \frac{(u - u_1)(u - u_2)}{k(u - u_m) + u_m}} \qquad (12)$$

The pressure p, density ρ and temperature T are found respectively from (4) and the second Equation (2). When $u_m = 0$ we obtain the exact solution of the problem of one-dimensional steady flow of a compressible conducting gas in a transverse magnetic field.

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Translated by A.H.A.